Recapitulación

$$\frac{\partial}{\partial t}\rho + \nabla\cdot J = 0$$

$$J(\vec{r},t) = \frac{\hbar}{2mi}\left[\psi^*\nabla\psi - \psi\nabla\psi^*\right] = \frac{\hbar}{m}\operatorname{Im}\{\psi^*\nabla\psi\}$$

$$\overset{?}{=} \frac{1}{m}\operatorname{Re}\left\{\psi^*\frac{\hbar}{i}\nabla\psi\right\}$$

Evolución de valores esperados

$$\frac{d}{dt}\langle\hat{A}\rangle = \frac{1}{i\hbar}\langle[\hat{A},\hat{H}]\rangle + \left\langle\frac{\partial\hat{A}}{\partial t}\right\rangle$$

Para una cantidad física clásica $\mathcal{A}(\vec{r},\vec{p},t)$
$\hat{A} = \mathcal{A}(\hat{\vec{R}},\hat{\vec{P}},t)$

Sólo cuando $\mathcal{A}$ depende de t el término $\frac{\partial\hat{A}}{\partial t}$ importa.

Para $\hat{H} = \frac{P^2}{2m} + V(\vec{R}) \Rightarrow \frac{d}{dt}\langle\vec{R}\rangle = \frac{1}{m}\langle\vec{P}\rangle$

$\frac{d}{dt}\langle\vec{P}\rangle = -\langle\nabla V(\vec{R})\rangle$

Variables conjugadas y medidas compatibles

Medidas incompatibles.

$$|\psi\rangle \xrightarrow[x_0]{\text{medir } \hat{X} \text{ y obtener}} |x_0\rangle \xrightarrow{\text{medir } \hat{P} \text{ y obtener } p_0} |p_0\rangle$$

$$\psi(x) \longrightarrow \delta(x-x_0) \longrightarrow \langle x|p_0\rangle = \frac{1}{(2\pi\hbar)^{1/2}}e^{\frac{i}{\hbar}p_0x}$$

$$\psi(p) \longrightarrow \langle p|x\rangle = \frac{1}{(2\pi\hbar)^{1/2}}e^{-\frac{i}{\hbar}px_0} \longrightarrow \delta(p-p_0)$$

Al medir $\hat{P}$ se borra la información que tenemos de $\hat{X}$.

Medidas compatibles

Medir $\hat{A}$ y obtener a_n proyecta al e-espacio de a_n.
Si tenemos otro observable $\hat{B}$ cuyos e-espacios están contenidos en los de $\hat{A}$ podemos medir $\hat{B}$ sin sacar al sistema del e-espacio de a_n así que al medir $\hat{A}$ de nuevo obtenemos a_n otra vez.

Ejemplo $\hat{A} = \left(|0\rangle\langle 0| + |2\rangle\langle 2|\right) - \left(|1\rangle\langle 1| + |3\rangle\langle 3|\right)$

$\hat{B} = \left(|0\rangle\langle 0| + 2|1\rangle\langle 1| + 3|2\rangle\langle 2| + 4|3\rangle\langle 3|\right)$

Medir $\hat{A}$ proyecta a pares / impares
Medir $\hat{B}$ no te saca de ahí

Teorema importante: $[A, B] = 0$ sii es posible construir una base ortonormal de e.v. comunes a A y B.

— Podemos medirlos simultáneamente.
Es decir, medir alguno de ellos no borra la información del otro

Consideremos $[\hat{A}, \hat{B}] = 0 \rightarrow$

distinguir

$$\hat{A}|a_n b_p, i\rangle = a_n |a_n, b_p, i\rangle$$
$$\hat{B}|a_n, b_p, i\rangle = b_p |a_n, b_p, i\rangle$$

¿Cuál es $P(a_n, b_p)$?

primero luego

$$P(A|B)=P(A), P(A|B)=\frac{P(A\cap B)}{P(B)} \to P(A\cap B)=P(A|B)P(B) \to P(A\cap B)=P(A)P(B)$$

Consideremos $|\psi\rangle = \sum_{n,p,i} c_{n,p,i}|a_n, b_p, i\rangle$

Si medimos $\hat{A}$

$$\mathcal{P}(a_n)=\sum_{p,i}|c_{n,p,i}|^2 \tag{C-32}$$

When we then measure B, the system is no longer in the state $|\psi\rangle$ but, if we have found a_n, in the state $|\psi'_n\rangle$:

$$|\psi'_n\rangle = \frac{1}{\sqrt{\sum_{p,i}|c_{n,p,i}|^2}}\sum_{p,i} c_{n,p,i}|a_n, b_p, i\rangle \tag{C-33}$$

The probability of obtaining b_p when it is known that the first measurement has yielded a_n is therefore equal to:

$$\mathcal{P}_{a_n}(b_p)=\frac{1}{\sum_{p,i}|c_{n,p,i}|^2}\sum_i |c_{n,p,i}|^2 \tag{C-34}$$

The probability $\mathcal{P}(a_n, b_p)$ sought corresponds to a "composite event": to be in a favorable case, we must first find a_n and then, having satisfied this first condition, find b_p. Therefore:

$$\mathcal{P}(a_n, b_p)=\mathcal{P}(a_n)\times\mathcal{P}_{a_n}(b_p) \tag{C-35}$$

Substituting into this formula expressions (C-32) and (C-34), we obtain:

$$\mathcal{P}(a_n, b_p)=\sum_i |c_{n,p,i}|^2 \tag{C-36}$$

Moreover, the state of the system becomes, immediately after the second measurement:

$$|\psi''_{n,p}\rangle \frac{1}{\sqrt{\sum_i |c_{n,p,i}|^2}}\sum_i c_{n,p,i}|a_n, b_p, i\rangle \tag{C-37}$$

Therefore, if we decide to measure either A or B again, we are sure of the result (a_n or b_p): $|\psi''_{n,p}\rangle$ is an eigenvector common to A and B with the eigenvalues a_n and b_p respectively.

Tarea:

$$\mathcal{P}(a_n, b_p)=\mathcal{P}(b_p, a_n)=\sum_i |c_{n,p,i}|^2=\sum_i |\langle a_n, b_p, i|\psi\rangle|^2$$

Ejemplos

$[R_i, R_j]=0 \qquad [P_i, P_j]=0 \qquad [R_i, P_j]=i\hbar\delta_{ij}$

$|x, y, z\rangle \qquad |P_x, P_y, P_z\rangle \qquad |x, P_y, z\rangle$

$|x, y, P_z\rangle$